Kant's question and debates about the boundaries of logic

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1 Introduction

Is math reducable to logic? Well, certain portions of mathematics have been shown to be reproduceable in certain formal systems. But do these systems really count as *logic* or is e.g. second order logic really "set theory in sheeps clothing" as Quine charmingly put it? Debates about what's the 'right' criterion for demarcating logic from the rest of mathematics (or the rest of a priori reasoning about necessary truths in general) can be frustrating and intractable. Whether "math reduces to logic" obviously depends on what you mean by logic. And many different distinguishing marks have been offered. Do you mean to ask whether math is reconstrctable in a system which is a) complete b) recusively axiomatizable c)"fully general" - whatever that means d) not possible to meaningfully doubt, because no more basic set of reasoning principles can be contrasted with it d) captures truth "in virtue of concepts" or what?

One good response to this situation would be to ramsify the word "logic" out of this debate. Decide which of the above mentioned properties you care about and then just go and investigate whether math is reduceable to something with that property without being confused into vainly debating whether your reduction base 'really' is or isn't logic with people who simply mean something different by the term. But perhaps none of these specifications really grabs you. Perhaps none of these questions 'Is math reducable to a formal system with specific property P?' seems to directly capture what you want to know when you ask yourself whether math is reduceable to logic. If so I have a tentative diagnosis and a suggestion, both which I will argue for in the rest of the paper.

The diagnosis is this. Kant's question 'How is synthetic a priori knowledge' and his radical answer to it made it seem like something could be gained by showing that mathematical truths were analytic. One wouldn't need to posit that there was anything less than-perfectly-objective about math, and one wouldn't need the facutly of pure intuition which many have found occult. So this might be (at some level) what you have in mind when you wonder about whether

Logicism suceeds in a way that can't be captured by any of the more specific questions above.

And if this diagnosis fits then, my suggestion is that you need to start investigating logicism by being a little patient and asking yourself a different question first - one which (to my knowledge) has not yet figured much in debates about logicism. The question is: how do you think *analytic* a priori knowledge is possible? Many people feel that it's somehow less mysterious that we can know things a priori if these things 'aren't really substantive' but this feeling isn't itself a story about how we can get (perfectly objective, naturalistically unproblematic etc.) knowledge of analytic truths. If you had such a story then it would be easy to figure out whether any given reduction of math to a formal system vindicated logicism. All you need to do is determine whether the mechanisms in your story can account for knowledge of the reduction base in questions. Figure out how you think we can know analytic truths and then see whether the putative 'logic' which math has been reduced to is knowable in this way.

2 Kant's Question

Hopefully the view-in-a-nutshell above made sense. Now I'll try to go through things more carefully.

Kant famously asked how synthetic a priori knowledge is possible. How come we can sit in our armchairs and come to conclusions in advance of experience, which experience then (seems to) conform to? And he gave math as an example to show that such knowledge must indeed be possible. In back of this question is a bigger question and an assumption. The big question is 'how is *any* a priori knowledge possible?' and the assumption is that we can non-problematically account for how merely analytic a priori knowledge is possible. Kant says remarkably little on how we are supposed to be able to get analytic knowledge [1].

One might motivate the big question along Kantian lines as follows: how can we sit on our armchairs and come to conclusions which experience then doesn't then contradict? How come I accept a priori stuff that then turns out to be true (whether trivially or not) like 'everything is self identical' rather than stuff I immediately find is false like 'everything is lime green'? And one might motivate the assumption by saying the fact that analytic truths are 'a mere matter of definition' and/or 'don't really rule anything out' should somehow help explain how we can get to these claims without using experience (we're just giving definitions) and then no future experience conflicts with them (they don't actually rule anything out). FYI the above sentence is total chicanery, but let's pretend we accept the assumption and move on.

Now Kant thinks the case of synthetic a priori knowledge where we can't diffuse the above question by appeal to analyticity is puzzling. He gives math

[1] See Michael Potter 'Reasons' Nearest Kin'

as an example to show that (somehow) there is such a thing as synthetic a priori knowledge. And then, as we know, he posits some very substantive metaphysics to explain how such synthetic a priori knowledge is possible. Our minds help construct the spatiotemporal world of experience. We have a faculty of intuition which lets us notice certain structural features which we always wind up constructing experience so as to have. This is faculty of intuition is then what lets us come up with principles in the armchair which all future experience will obey. But those principles aren't fully objectively true (as shown by the fact that they lead to paradox when applied to certain situations). Rather, they reflect artifacts of the way that we (and perhaps all creatures with minds even loosely like ours) construct the world.

So we have the mystery of learning things from the armchair and two explanations of it corresponding to the two different kinds of things we can learn. One explanation is assumed (you can learn that everything is identical from the armchair because it's just a matter of definition/isn't really saying anything). And the other is Kant's proposal.

3 Logicism

However, this ingenious proposal of Kant's comes with a high price. First we have to say that there's some sense in which mathematical claims - which one would have thought were paradigms of objectivity- aren't fully objective. Second we have to invoke the faculty of rational intuition which many people find just as puzzling as the original possibility of substantive a priori knowledge. I mean there are lots of things that are true of all my actual experiences - how do I tell which ones are necessary artifacts of the way my mind constructs the world vs. mere contingencies of what my noumenal input happens to have been so far? Do I just try imagining different scenarios? But someone who was raised with color to black and white contact lenses might well find visual differences in experience without greyscale differences inconceivable - yet presumably they would not be having a rational intuition that color experiences were impossible.

Kant himself gives long arguments in favor of his various claims that are supposed to be rational intuitions. So this suggests that you are supposed to reason to the conclusion that something is not just a feature of your current experience and all the thought experiments you have actually done so far. But what's the nature of this reasoning? Is it a mere matter of logically unpacking definitions (and hence knowable however analytic truths are supposed to be known)? Or does this reasoning itself involve further rational intuitions i.e. conclusions you have already drawn about how all future experience must be constructed? But then how do you get the first non-analytic conclusion about how all experience must be?

So, you might well not like to find yourself in bed with Kantian intuition, or having to say that statements like, 'There are infinitely many primes' are somehow subjective. I claim this is a big motive for interest in Logicism. If you can show that (some portion of) math is really analytic you don't need to say it (/that portion of it) is subjective. And if you can say that all math is analytic you have knocked out Kant's motivating example to show that there is such a thing as synthetic a priori knowledge. Hence you would remove the question which rational intuition had to be posited to solve. But is math analytic? Can all math be arrived at by logic and definitions alone? This is the task to be attempted - and getting rid of spooky rational intuition and idealism about algebra will be the prize if we succeed.

Now you open up a modern book on philosophy of math and you see that all math can be reconstructed in set theory, which itself can be reconstructed in second order logic. Second order logic is, after all called "logic". So do you declare victory or what? You might try to conceptually analyze the word "logic" by coming up with a definition that would give up a verdict in this case but a) conceptual analysis has a terrible track record and b) is it really clear that there is one special feature among the many interesting formal properties which putative 'logics' can have which should be honored as the real criterion for logicality?

In this paper I have tried to suggest a build up the case for a different, more constructive response. If what you care about is (ultimately) whether mathematics can be known in the fully objective naturalistically unproblematic way that you think anlaytic truths/matters of "logic" and definitions can be then what you should do is go back and think about what the 'way' in question is. Once you have figured out what kinds of mechanisms you think could give us (fully objective, naturalistically understandable) analytic knowledge then you can just see whether these mechanisms could also let you discover second order logic and hence set theory and all of mathematics.

4 Conclusion

So, the net result I have argued for is this. If you care about Logicism (or directly about getting rid of Kantian intuition or idealism about philosophy of math), and you want to know how various mathematical results bear on this question, you need to step back and look at the bigger picture of epistemology of the a priori. Only if you can get an explicit grasp on how *"analytic"* truths are supposed to be knowable a priori can you determine whether the same explanation applies to second order logic - and hence whether reducing math to second order logic allows you to sidestep the problem that Kantian intuition is supposed to solve.

COMMERCIAL. As people living after Quine, we might not want to phase the above question in a way that *assumes* there is a principled distinction between analytic and synthetic truths, or truths of logic and truths of mathematics or even science. So if you remove the commitment to a principled distinction between analytic and synthetic from the above you get the big question I claimed was in the background for Kant 'How is *any* a priori knowledge possible?'. As it happens, my thesis proposes an answer to (one popular version of) this question. Check it out at http://www.people.fas.harvard.edu/ seberry/