The problem of a priori *analytic* knowledge

Sharon Berry

March 25, 2009

Abstract

Many people who find our knowledge of mathematics puzzling, find our knowledge of logic and analytic truths much less puzzling. Somehow, the fact that statements like 'All bachelors are unmarried' are supposed to be 'just part of what we mean by the word 'bachelor" in a way that explains our ability to learn such truths without appeal to experience. In contrast, our apparent knowledge of more 'substantive' mathematical knowledge remains mysterious. We seem to have a kind of unproblematic access to analytic truths (we can learn them just by "inspecting our concepts") which does not apply to analytic truths. In this paper, I will argue that this impression is false. If we actually have some kind of faculty of concept inspection which gave us unproblematic access to analytic truths this would straightforwardly give us a way of getting supposedly 'substantive' mathematical knowledge as well. Easy access to analytic truths is a myth.

1 Introduction

Many people who find our knowledge of mathematics puzzling, find our knowledge of logic and analytic truths much less puzzling. Somehow, the fact that statements like 'All bachelors are unmarried' are supposed to be

’just part of what we mean by the word ’bachelor” in a way that explains our ability to learn such truths without appeal to experience. In contrast, or apparent a priori knowledge of more ’substantive’ matters - like claims about what is provable or what is consistent - remains mysterious. We seem to have a kind of unproblematic access to analytic truths (we can learn them just by ”inspecting our concepts”) which does not apply to analytic truths. In this paper, I will argue that this impression is false. If we actually have some kind of faculty of concept inspection which gave us unproblematic access to analytic truths this would straightforwardly give us a way of getting supposedly ’substantive’ mathematical knowledge as well. Easy access to analytic truths is a myth.

2 Concept inspection

Let’s start by laying out and the intuitive idea that I want to argue against (and motivating it a little, though I don’t think it needs much motivation). The idea is this: we have a distinctive method for learning analytic truths which does not yield knowledge of synthetic truths, which one might call concept inspection.

Concept Inspection: Just by ’inspecting our concept’ of bachelorhood we can reliably form true beliefs like ’all bachelors are unmarried’, and acquire truth-preserving inference mechanisms like inferring sentences of the form ’x is not a table’ from sentences of the form ’x is a bachelor’. One sees that these sentences are true, and these inference methods are truth preserving just by attending to what the word ’bachelor’ means. And, as an english speaker you have unproblematic access to trivial/analytic facts of that kind. However, this easy route cannot lead to knowledge of more substantive claims which are not ’mere matters of definition’.

This common intuition that concept inspection gives us unproblematic a priori knowledge, has a little to do with the idea of stipulative definition. Suppose I stipulate that I will be using "bachelor" to mean unmarried man, and then I say "For all x, if x is a bachelor then x is an unmarried men". You might think that my antecedent stipulation has guaranteed that the above sentence will -in my mouth at least- express a truth. So here I have a nice reliable method for producing true beliefs: make a stipulation, and then assert sentences which this stipulation ensures will express a truth (and form the corresponding belief). However, corresponding to the ease of getting this knowledge there's a kind of arbitrariness and triviality to the facts which I have thus learned: they just fall out of our arbitrary choice of stipulative definitions.

Now, concept inspection doesn't exactly work like this: we don't remember back to some point at which we explicitly made a stipulation before claiming that anything is an analytic truth. But (intuitively) our ability to learn things a priori by concept inspection is supposed to work in essentially the same way.

3 Why a faculty of concept inspection would yield mathematical knowledge as well as analytic knowledge

Now I claim: If we have the above mentioned faculty of concept inspection, it would also suffice to yield substantive mathematical knowledge.

For, suppose that applying my faculty of concept inspection to my concept 'horse' reliably tells me that the truth of certain sentences S1...Sn and the validity of certain (syntactically characterized) inference methods I1...Im (probably ones that that connect different propositions about horses) is just part of what we mean by 'horse'. Then I can reliably infer that a certain formal system is consistent. Why? Applying truth preserving inference methods to true sentences can only yield true statements. So, it cannot be possible to prove the falsehood "0=1" in the formal system which takes the sentences S1...Sn as axioms and the syntatically characterized I1...Im as permissible inference methods.

But this knowledge of consistency facts, what is provable in a given formal system is a paradigmatic example of the kind of 'substantive' facts which concept inspection is *not* supposed to be able to give us. We don't just understand our notion of consistency in such a way that consistency means whatever it needs to mean to make this jumble of sentences and inferences about horses which we find compelling wind up counting as 'consistent'. It's an infinitary statement about what you can get by reapplying the inference rules arbitrarily many times. It's (mathematically) equivalent to a statement of the the form $\forall$ x Fx (namely, 'no number is the godel number of a proof of "0=1" in this formal system), which quantifies over numbers, and cannot be verified by checking any finite number of cases. It seems to be a substantive, objective fact - not the kind of thing we can fix by stipulation.

So, if we could really reliably come to form true beliefs by stipulation/concept inspection like 'all bachelors are unmarried' this would directly allow us to learn other things which do *not* seem to be a matter of stipulation or convention like 'such and such a formal system is consistent'?

4 Analytic knowledge doesn't come for free

What's going on? This whole process of getting substantive mathematical facts out of arbitrary stipulation/ facts about our concepts may seem like spinning straw into gold.

Here's the deal. It seems like we are free to arbitrarily stipulate: 'I mean to use my words in such a way that such-and-such sentences are true and so-and-so inferences are valid'. Then, given the semantic facts fixed by these stipulations, certain sentences are bound to express truths - and we can exploit this fact to arrive at new true beliefs, by concocting and then accepting such sentences. This gives us a way of producing a priori knowledge - though the knowledge it yields may seem somehow trivial. This (I think) is what gets the intuition that concept inspection gives us unproblematic access to analytic truths going.

But in fact, we **aren't** free to make arbitrary stipulations in this way. Making arbitrary stipulations that you intend to use your words in such a way that such-and-such sentences wind up being true and so-and-so inference methods wind up being valid is actually *not at all a reliable way of forming true beliefs*! For, if the stipulations you make aren't (at least) consistent, there will be no candidate meaning available for the word which you claim to be implicitly defining to take on. A traditional example of this is the pseudo truth-functional connective 'tonk' which is supposed to have the introduction rules for 'or' and the elimination rules for 'and'.

But now this consistency requirement means that, *to the extent that you can* reliably form true beliefs by stipulative definition or reflecting on what claims are 'just part of what we mean by the words' you must already be correctly responding to non-trivial, non-stipulative mathematical facts about consistentency. Reflecting on 'what's just part of our concept' or making stipulative definitions can only reliably lead us to form true beliefs if we can (implicitly or explicitly) spot and avoid inconsistent stipulations/incoherent concepts like tonk. In using the concept inspection and stipulative definition to form true beliefs, one is implicitly using some kind of mathematical knowledge about consistency (usually of a fairly simple kind) - or some kind of stronger faculty of philosophical insight which gives one access to mathematical facts and something else as well - to check that the stipulations are at least consistent.

5 Consequences

The previous paragraph is the heart of this paper, so let me dwell more on what I think it shows.

Many people find it un-mysterious that we seem to be able to reliably form true beliefs like 'bachelors are unmarried', but find our ability to reliably get the right answers about more substantive mathematical/ a priori matters like consistency facts (which they don't want to say are 'just part of what we mean by consistency') puzzling. The latter seems to require a mysterious faculty of rational intuition. But this popular combination of attitudes is un-teniable.

If we are able to reliably form true beliefs by making stipulative implicit definitions, then we are also able to reliably come up with consistent rather than inconsistent stipulations - because unless you can avoid making inconsistent (tonk-like) stipulations, the word you are defining won't be meaningful and the sentences you wind up accepting at won't express truths. If inspecting our concepts by stating what seem to us like obvious conceptual truths about Xs is a reliable method of forming true beliefs, then we must somehow manage to avoid finding inconsistent statements to be 'obvious conceptual truths'.

You can't be puzzled about how we are able to track consistency facts at all, but not puzzled about how we have a faculty of concept inspection which allows us to reliably form analytically true beliefs because **the only way a faculty of concept inspection can be reliable is if it is sensitive to consistency facts**. If you can't either avoid generating, or spot and toss out bad combinations of putative conceptual truths like those corresponding to 'tonk' you don't have a reliable method of forming beliefs.

But now, suppose you admit that in forming beliefs about what's an analytic truth you can fairly reliably avoid making inconsistent stipulations, or coming to feel that inconsistent statements are obvious conceptual truths. So far (I admit) we don't have any explicit knowledge about consistency, because we don't even have any beliefs about consistency. It's one thing to reliably make consistent stipulations. It's another think to have knowledge that certain stipulations are consistent. I might make consistent stipulations without know it. But suppose I actually do go through the reasoning at the beginning of this section, and thus form the belief that the sentences (and inference methods) in question are consistent. Then (it certainly seems) that I have learned that the sentences in question are consistent.

Thus, it would seem that any faculty of concept inspection which reliably generated true beliefs about analytic statements would also suffice to give us knowledge of paradigmatically substantive, non-arbitrary matters like consistency facts as well.

6 Conclusion

So, I have argued that we don't have some kind of special access to analytic truths which is less puzzling than our access to apparently non-analytic matters like consistency facts. In order to arrive at truths by inspecting what seems to be (or what you are stipulating to be) 'just part of the concept of xs' you need to do more than pick arbitrary stipulations and follow through on them. You need to successfully hone in on combinations of stipulations which have certain non-trivial mathematical properties like consistency.

This may seem like a depressing conclusion: I doubt any one who accepts my argument will respond to the news that the kind of faculty we'd need to reliably get true beliefs about analytic statements is one which would also yield knowledge of consistency statements by becoming less worried about knowledge of consistency statements!

Rather, the upshot of the above argument (if you buy it) is to increase the range of the mystery about how substantive a priori knowledge is possible. We thought that at least some portion of a priori knowledge - knowledge of the analytic- was unproblematic, but now it turns out that we need to posit the same kind of substantive faculty of rational intuition which seemed so puzzling in the case of mathematical knowledge, in order to account for our ability to get analytic knowledge as well.

But, so as not to end on a gloomy note, let me briefly (and I hope not *too* cryptically) point in the direction of a silver lining. If the above reasoning works 'all these statements and inference methods are conceptual truths, so they are truths, so they are consistent' it also works for *empirical* statements and inference methods. So if you can learn empirically that certain, say, medical sentences are true, and certain (syntactically characterized) medical inference methods are exceptionlessly truth-preserving then you can parley this into the knowledge that the corresponding formal system is consistent.